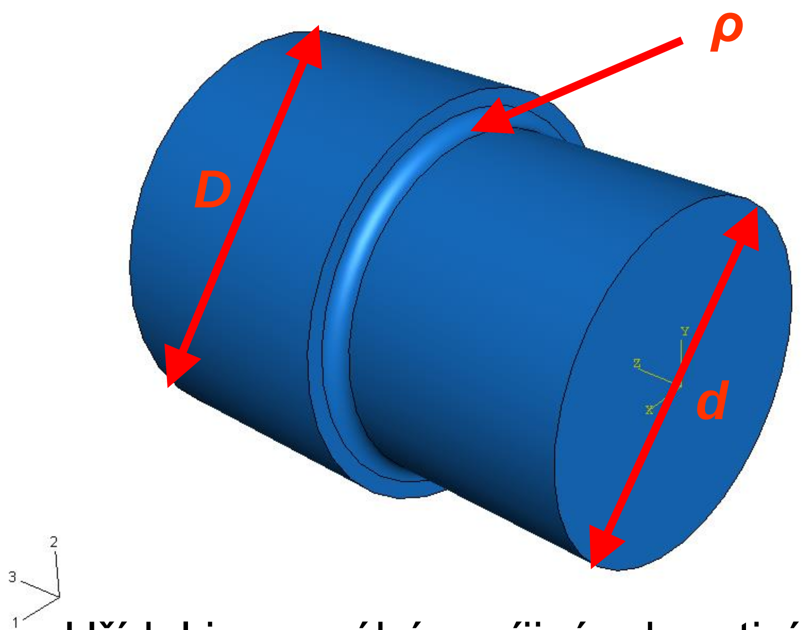
# Dynamická pevnost a životnost
# Cvičení

Milan Růžička, Josef Jurenka,
Martin Nesládek, Jan Papuga

mechanika.fs.cvut.cz martin.nesladek@fs.cvut.cz

# Hřídelové osazení

# Zadání




$D = 48$ mm

$d = 40$ mm

$\rho = 2$ mm

$P = 100$ kW, $n = 1500$ $\min^{-1}$,

$M_o = 200000$ N.mm

ocel 12040:

$R_m = 700$ MPa

$R_{p0,2} = 560$ MPa

soustruženo: *Ra*=1,6

Hřídel je namáhán míjivým krouticím momentem a symetricky střídavým ohybem. Stanovte bezpečnost vůči mezi únavy.

Jsou dány meze únavy pro ohyb (300 MPa) a krut (175 MPa) a také pro tah-tlak (245 MPa)

Oba silové účinky působí ve fázi
=
nejdestruktivnější kombinace
*Mo* a *Mk*

# Namáhání (menší průřez)

$$M_o = 200000 \quad \text{N.mm}$$

$$M_{om} = 0 \ \text{N.mm} \qquad M_{oa} = 200000 \quad \text{N.mm}$$

$$W_o = \frac{\pi d^3}{32} = \frac{\pi 40^3}{32} = 6283 \quad \text{mm}^3$$

$$\sigma_{oa} = \frac{M_{oa}}{W_o} = \frac{32 M_{oa}}{\pi d^3} = \frac{32 \cdot 200000}{\pi 40^3} = 32 \ \text{MPa}$$

$$\sigma_{om} = 0 \ \text{MPa}$$

$$M_k = \frac{P}{\pi \frac{n}{30}} = \frac{100000}{\pi \frac{1500}{30}} = 636620 \quad \text{N.mm}$$

$$M_{km} = 318310 \quad \text{N.mm} \qquad M_{ka} = 318310 \quad \text{N.mm}$$

$$W_k = \frac{\pi d^3}{16} = \frac{\pi 40^3}{16} = 12566 \quad \text{mm}^3$$

$$\tau_a = \frac{M_{ka}}{W_k} = \frac{16 M_{ka}}{\pi d^3} = \frac{16 \cdot 318310}{\pi 40^3} = 25{,}3 \ \text{MPa}$$

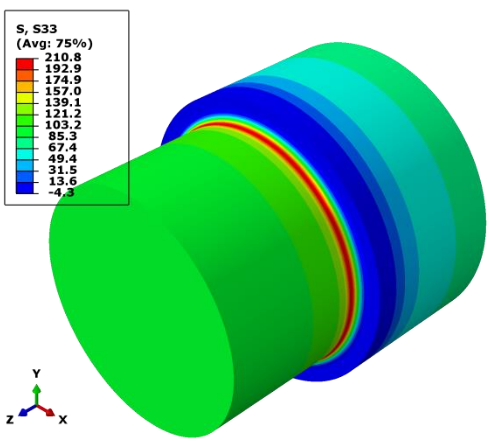
$$\tau_m = \frac{M_{km}}{W_k} = \frac{16 M_{km}}{\pi d^3} = \frac{16 \cdot 318310}{\pi 40^3} = 25{,}3 \ \text{MPa}$$

# Odhady meze únavy

$$\sigma_c = 245 \quad \text{MPa}$$

$$\eta_p \cong 0{,}85$$

$$\varepsilon_v = 0{,}83$$

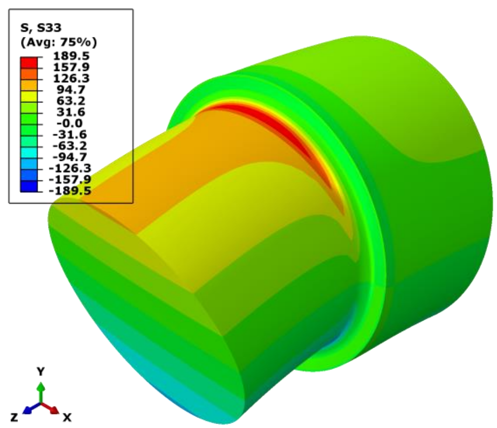



$$\alpha = 2{,}108$$

$$\sigma_{co} = 300 \quad \text{MPa}$$

$$\eta_{po} \cong 0{,}85$$

$$\varepsilon_{vo} \cong 0{,}83$$

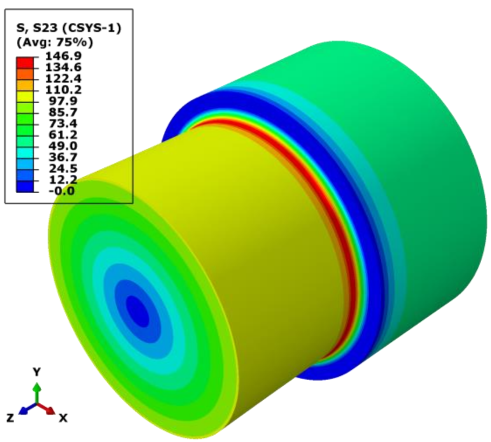



$$\alpha_o = 1{,}895$$

$$\tau_c = 175 \quad \text{MPa}$$

$$\eta_{pk} \cong \frac{1}{2}\left(1 + \eta_{po}\right) = \frac{1}{2}\left(1 + 0{,}85\right) = 0{,}925$$

$$\varepsilon_{vk} \cong 0{,}83$$




$$\alpha_k = 1{,}469$$

Součinitele tvaru určeny pro nominální napětí 100 MPa.
Různé způsoby určení součinitele vrubu…

# Součinitel vrubu - ohyb

| Thum | Peterson | Neuber | Heywood |
|---|---|---|---|
| $\beta_o = 1 + (\alpha_o - 1) q_o$ | $\beta_o = 1 + \dfrac{\alpha_o - 1}{1 + \dfrac{a}{\rho}}$ | $\beta_o = 1 + \dfrac{\alpha_o - 1}{1 + \sqrt{\dfrac{A}{\rho}}}$ | $\beta_o = \dfrac{\alpha_o}{1 + \dfrac{\alpha_o - 1}{\alpha_o} \dfrac{p}{\sqrt{\rho}}}$ |
| $q_o = 0{,}76$ | $a \cong 0{,}0254 \left(\dfrac{2070}{Rm}\right)^{1{,}8}$ | $\sqrt{A} = 0{,}3$ | $p = \dfrac{280}{\sigma_{pt}}$ |
| $\beta_o = 1{,}68$ | $\beta_o = 1{,}82$ | $\beta_o = 1{,}73$ | $\beta_o = 1{,}67$ |

V situaci, kdy by nebylo k dispozici doporučení či vztah používaný pro dané průmyslové odvětví, je nutné učinit více odhadů a konzervativně volit nejvyšší „beta“.

$$\sigma_{co}^{x} = \frac{\sigma_{co} \eta_{po} \varepsilon_{vo}}{\beta_o} = \frac{300 \cdot 0{,}85 \cdot 0{,}83}{1{,}82} = 116 \text{ MPa}$$

# Součinitel vrubu - krut

| Thum | Peterson | Neuber | Heywood |
|---|---|---|---|
| $\beta_k = 1 + (\alpha_k - 1) q_k$ | $\beta_k = 1 + \dfrac{\alpha_k - 1}{1 + \dfrac{a}{\rho}}$ | $\beta_k = 1 + \dfrac{\alpha_k - 1}{1 + \sqrt{\dfrac{A}{\rho}}}$ | $\beta_k = \dfrac{\alpha_k}{1 + \dfrac{\alpha_k - 1}{\alpha_k} \dfrac{p}{\sqrt{\rho}}}$ |
| $q_k = 0{,}83$ | $a \cong 0{,}25 \div 0{,}3$ | $\sqrt{A} = 0{,}3$ | $p = \dfrac{280}{\sigma_{pt}}$ |
| $\beta_k = 1{,}39$ | $\beta_k = 1{,}42$ | $\beta_k = 1{,}39$ | $\beta_k = 1{,}35$ |

V situaci, kdy by nebylo k dispozici doporučení či vztah používaný pro dané průmyslové odvětví, je nutné učinit více odhadů a konzervativně volit nejvyšší „beta“.

$$\tau_c^x = \frac{\tau_c \eta_{pk} \varepsilon_{vk}}{\beta} = \frac{175 \cdot 0{,}925 \cdot 0{,}83}{1{,}42} = 94 \text{ MPa}$$

# Součinitel vrubu - tah-tlak

### Thum

$$\beta = 1 + (\alpha - 1)q$$

$$q = 0{,}76$$

$$\beta = 1{,}84$$

### Peterson

$$\beta = 1 + \frac{\alpha - 1}{1 + \frac{a}{\rho}}$$

$$a \cong 0{,}25 \div 0{,}3$$

$$\beta = 1{,}98$$

### Neuber

$$\beta = 1 + \frac{\alpha - 1}{1 + \sqrt{\frac{A}{\rho}}}$$

$$\sqrt{A} = 0{,}3$$

$$\beta = 1{,}91$$

### Heywood

$$\beta = \frac{\alpha}{1 + \frac{\alpha - 1}{\alpha}\frac{p}{\sqrt{\rho}}}$$

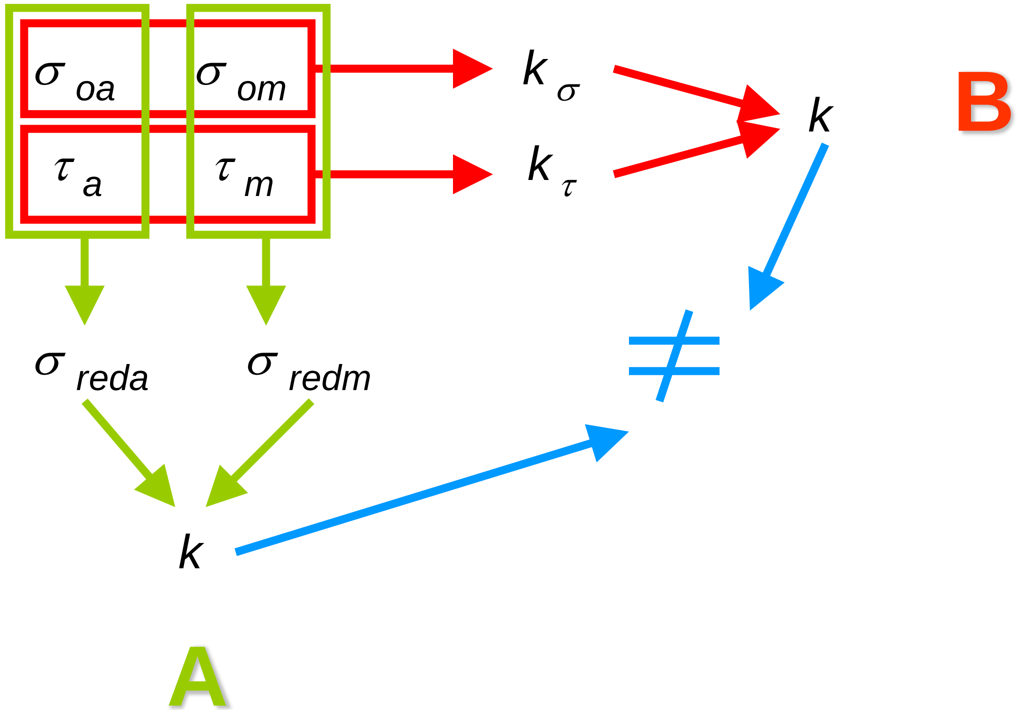
$$p = \frac{280}{\sigma_{pt}}$$

$$\beta = 1{,}83$$

V situaci, kdy by nebylo k dispozici doporučení či vztah používaný pro dané průmyslové odvětví, je nutné učinit více odhadů a konzervativně volit nejvyšší „beta“.

$$\sigma_c^x = \frac{\sigma_c \eta_p \varepsilon_v}{\beta} = \frac{245 \cdot 0{,}85 \cdot 0{,}83}{1{,}98} = 87 \text{ MPa}$$

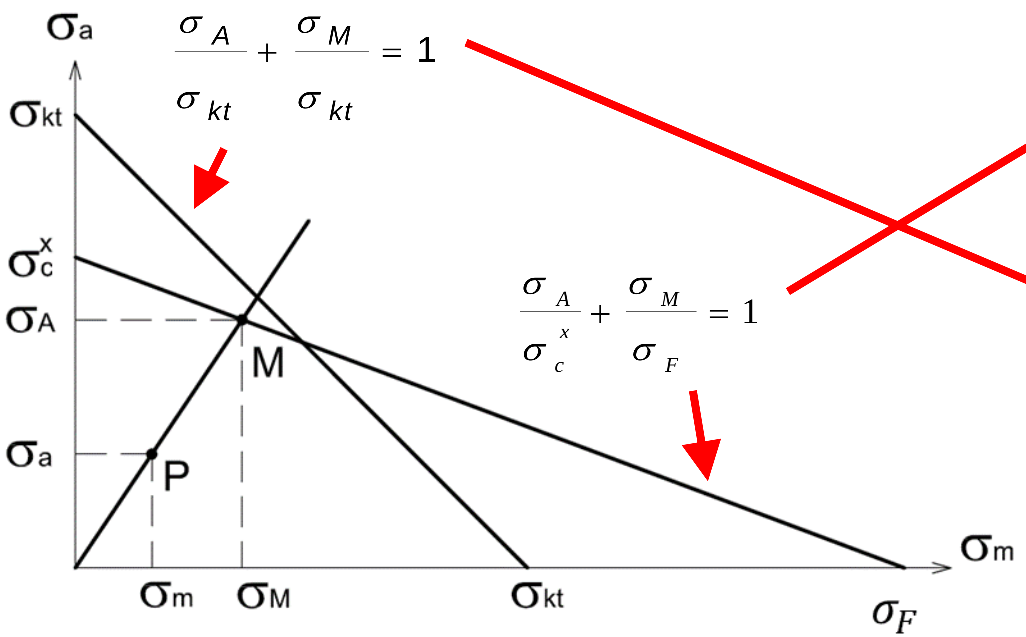
# Bezpečnost – různé přístupy…



# Haighův diagram

## (Goodman diagram)

$$\frac{\sigma_A}{\sigma_{kt}} + \frac{\sigma_M}{\sigma_{kt}} = 1$$




$$\frac{\sigma_A}{\sigma_c^x} + \frac{\sigma_M}{\sigma_F} = 1$$

$$k_1 = \frac{1}{\frac{\sigma_a}{\sigma_c^x} + \frac{\sigma_m}{\sigma_F}}$$

$$k_2 = \frac{1}{\frac{\sigma_a}{\sigma_{kt}} + \frac{\sigma_m}{\sigma_{kt}}}$$

$$k = \min(k_1, k_2)$$

$$\sigma_A = k\sigma_a \qquad \sigma_M = k\sigma_m$$

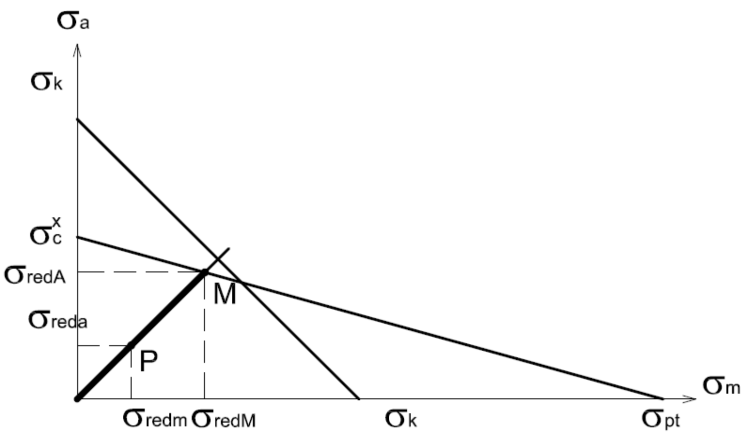
**odhad fiktivního napětí:**

tah: $\sigma_F = Rm$

ohyb: $\sigma_F = (1{,}5 \div 1{,}7)Rm$

krut: $\tau_F = (0{,}7 \div 0{,}8)Rm$

# A standard) Haighův diagram




$$\sigma_{reda} = \sqrt{\sigma_{oa}^{2} + 3\tau_{a}^{2}} = \sqrt{32^{2} + 3 \cdot 25{,}3^{2}} = 54{,}3 \text{ MPa}$$

$$\sigma_{redm} = \sqrt{\sigma_{om}^{2} + 3\tau_{m}^{2}} = \sqrt{0^{2} + 3 \cdot 25{,}3^{2}} = 43{,}8 \text{ MPa}$$

$$k_1 = \frac{1}{\frac{\sigma_{reda}}{\sigma_c^x} + \frac{\sigma_{redm}}{R_m}} = \frac{1}{\frac{54{,}3}{87} + \frac{43{,}8}{700}} = 1{,}49$$

$$k_2 = \frac{1}{\frac{\sigma_{reda}}{R_{p0,2}} + \frac{\sigma_{redm}}{R_{p0,2}}} = \frac{1}{\frac{54{,}3}{560} + \frac{43{,}8}{560}} = 5{,}71$$

**k**

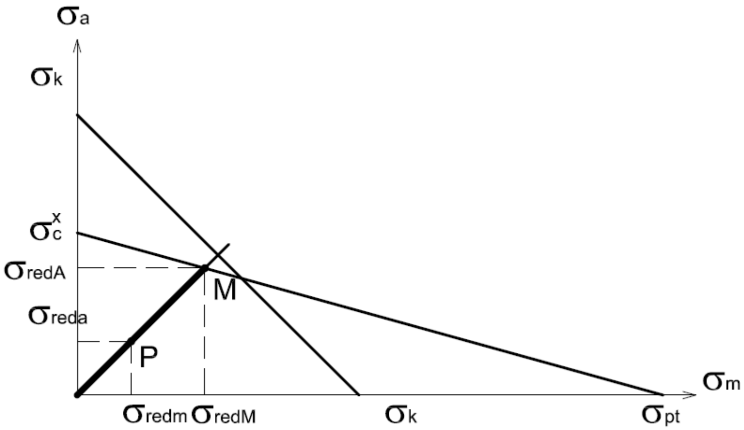
$$k = \min(k_1, k_2) = \min(1{,}49\,;\,5{,}71) = 1{,}49$$

# A přesnější) Haighův diagram

$$\sigma_{redh} = \sqrt{\sigma_{oh}^2 + 3\tau_h^2} = \sqrt{32^2 + 3\cdot 50{,}6^2} = 93{,}3\ \text{MPa}$$

$$\sigma_{redd} = \sqrt{\sigma_{od}^2 + 3\tau_d^2} = \sqrt{(-32)^2 + 3\cdot 0^2} = 32\ \text{MPa}$$

po signování $\sigma_{redd} = -32\ \text{MPa}$




$$\sigma_{reda} = \frac{\sigma_{redh} - \sigma_{redd}}{2} = \frac{93{,}3 - (-32)}{2} = 62{,}7\ \text{MPa}$$

$$\sigma_{redm} = \frac{\sigma_{redh} + \sigma_{redd}}{2} = \frac{93{,}3 + (-32)}{2} = 30{,}7\ \text{MPa}$$

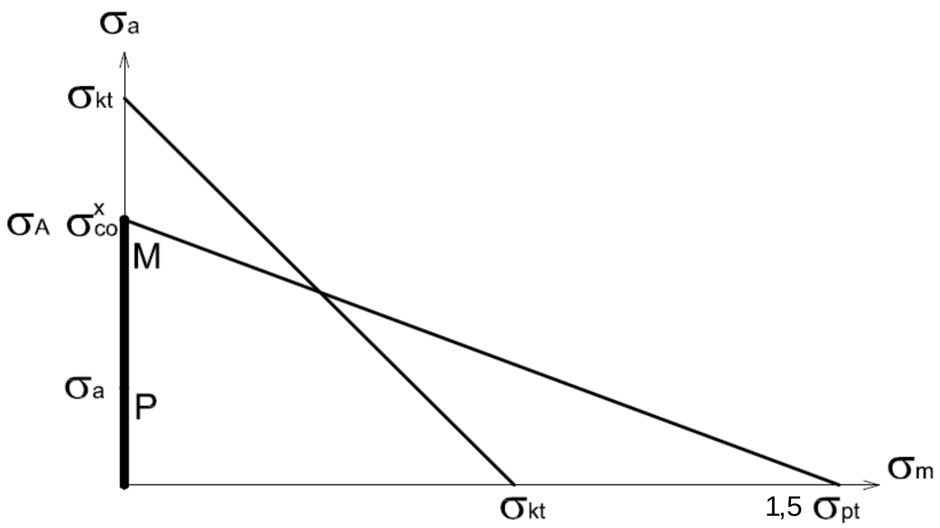
$$k_1 = \frac{1}{\dfrac{\sigma_{reda}}{\sigma_c^x} + \dfrac{\sigma_{redm}}{R_m}} = \frac{1}{\dfrac{62{,}7}{87} + \dfrac{30{,}7}{700}} = 1{,}31$$

$$k_2 = \frac{1}{\dfrac{\sigma_{reda}}{R_{p0,2}} + \dfrac{\sigma_{redm}}{R_{p0,2}}} = \frac{1}{\dfrac{62{,}7}{560} + \dfrac{30{,}7}{560}} = 6{,}00$$

**k**

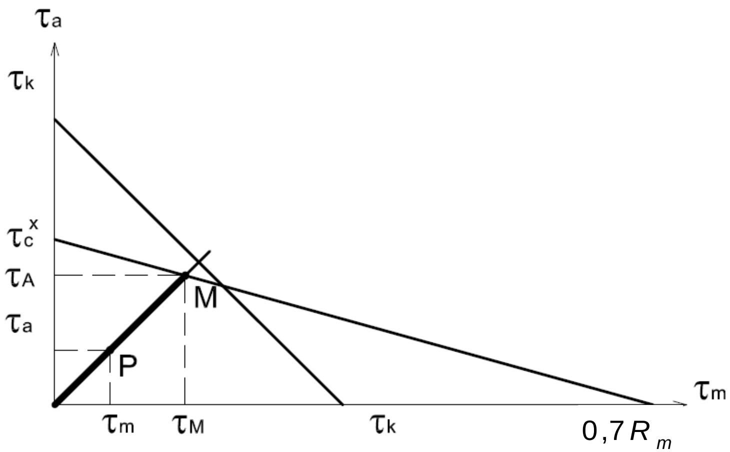
$$k = \min(k_1, k_2) = \min(1{,}31; 6) = 1{,}31$$

# B standard) Haighův diagram - ohyb




$$k_\sigma = \frac{\sigma^x_{co}}{\sigma_{oa}} = \frac{116}{32} = 3{,}63$$

# B standard) Haighův diagram - krut

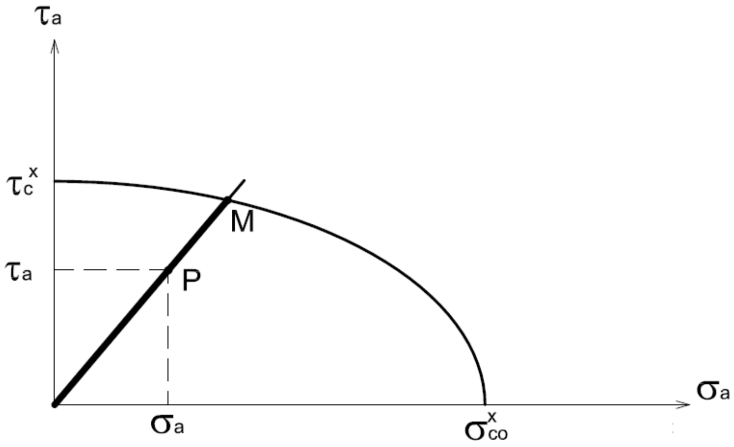



$$k_{\tau 1} = \frac{1}{\frac{\tau_a}{\tau_c^x} + \frac{\tau_m}{0{,}7 R_m}} = \frac{1}{\frac{25{,}3}{94} + \frac{25{,}3}{0{,}7 \cdot 700}} = 3{,}11$$

$$k_{\tau 2} = \frac{1}{\frac{\tau_a}{R_{p0,2}/\sqrt{3}} + \frac{\tau_m}{R_{p0,2}/\sqrt{3}}} = \frac{1}{\frac{25{,}3}{560/\sqrt{3}} + \frac{25{,}3}{560/\sqrt{3}}} = 6{,}39$$

$$k_{\tau} = \min\left(k_{\tau 1}, k_{\tau 2}\right) = \min\left(3{,}11;\ 6{,}39\right) = 3{,}11$$

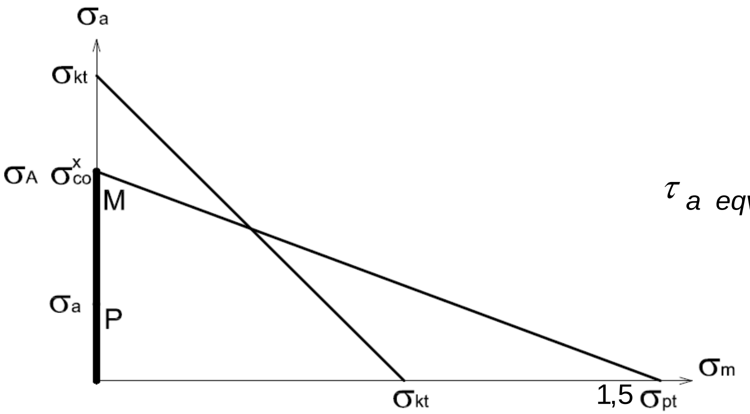
# B standard) Kombinace namáhání




k

$$\frac{1}{k^2} = \frac{1}{k_\sigma^2} + \frac{1}{k_\tau^2} \Rightarrow k = \sqrt{\frac{1}{\frac{1}{k_\sigma^2} + \frac{1}{k_\tau^2}}} = \sqrt{\frac{1}{\frac{1}{3{,}63^2} + \frac{1}{3{,}11^2}}} = 2{,}36$$

# B přesnější) Kombinace namáhání s ekvivalentní amplitudou napětí (…)




$$k_\sigma = \frac{\sigma_{co}^x}{\sigma_{oa}} = \frac{116}{32} = 3{,}63$$

$$\tau_{a\ eqv} = \sqrt{\tau_a(\tau_a + \tau_m)} = \sqrt{25{,}3(25{,}3 + 25{,}3)} = 35{,}77 \text{ MPa}$$

$$k_\tau = \frac{\tau_c^x}{\tau_{a\ eqv}} = \frac{94}{35{,}77} = 2{,}62$$

$$\frac{1}{k^2} = \frac{1}{k_\sigma^2} + \frac{1}{k_\tau^2} \Rightarrow k = \sqrt{\frac{1}{\frac{1}{k_\sigma^2} + \frac{1}{k_\tau^2}}} = \sqrt{\frac{1}{\frac{1}{3{,}63^2} + \frac{1}{2{,}62^2}}} = 2{,}12$$

**k**